Numerical Optimal Transport

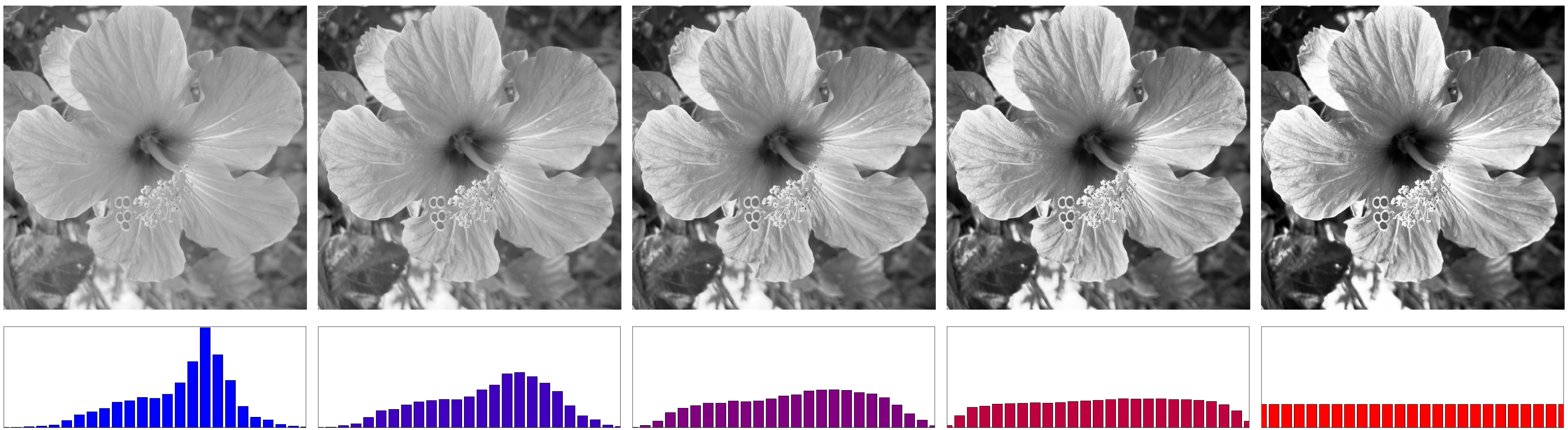
http://optimaltransport.github.io

Applications

Gabriel Peyré

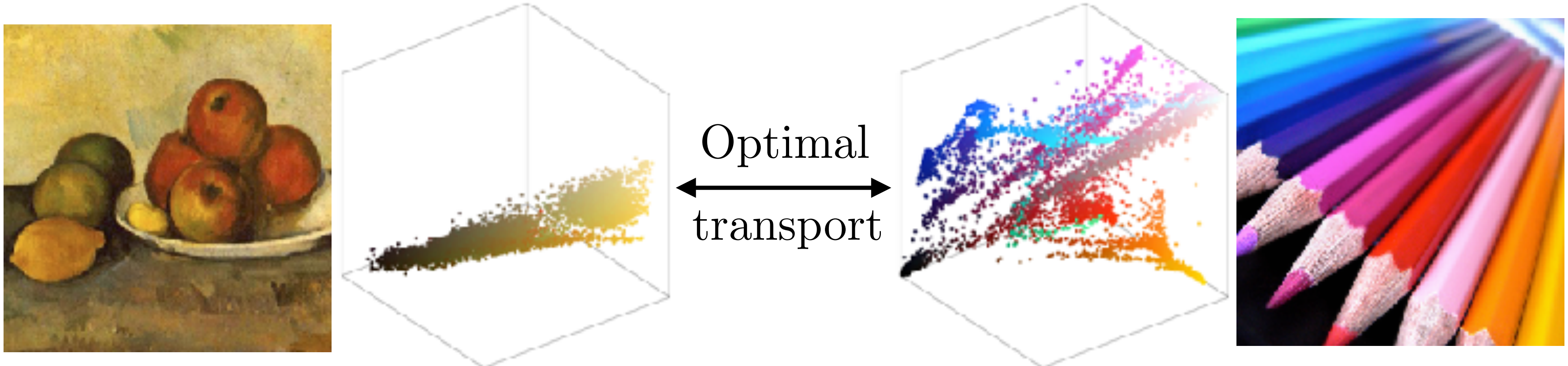
www.numerical-tours.com

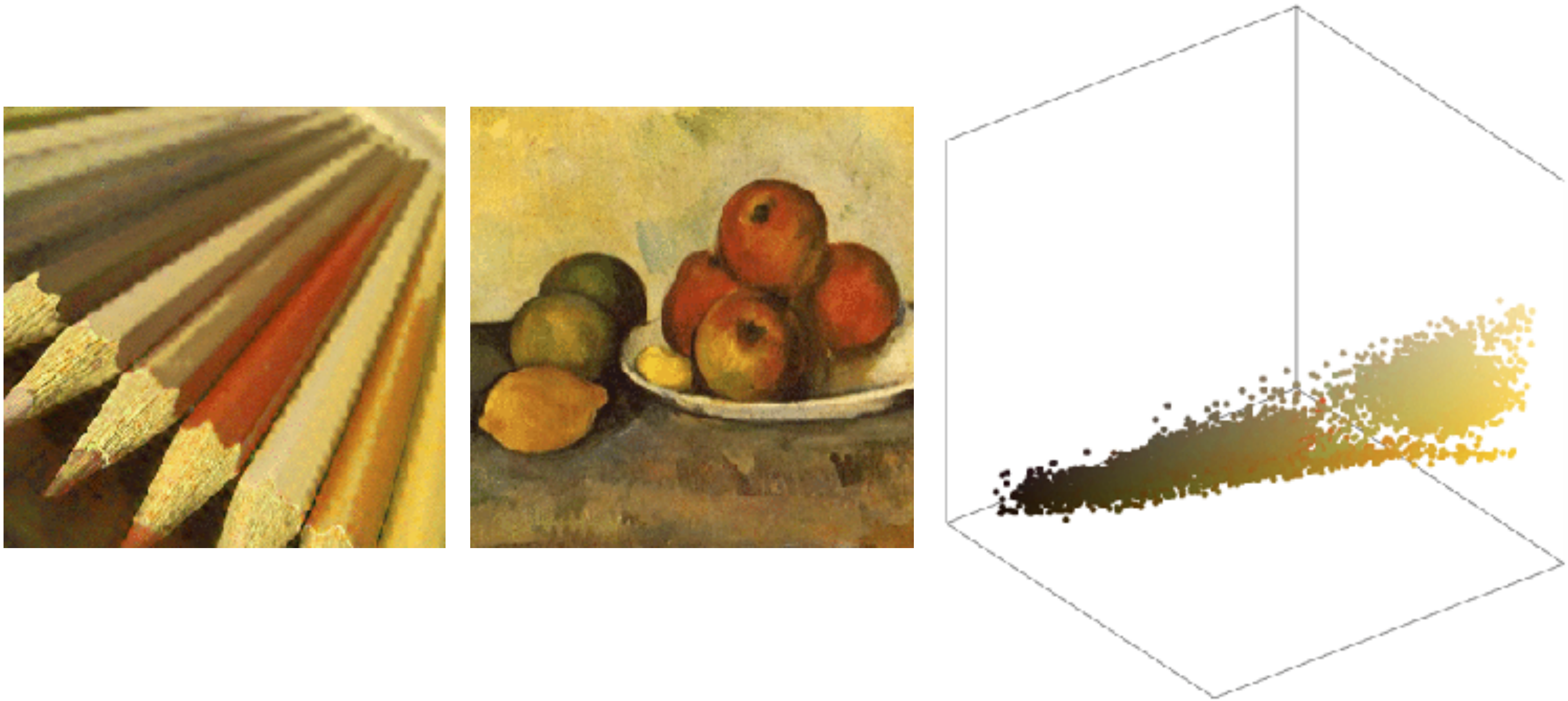
Grayscale Image Equalization



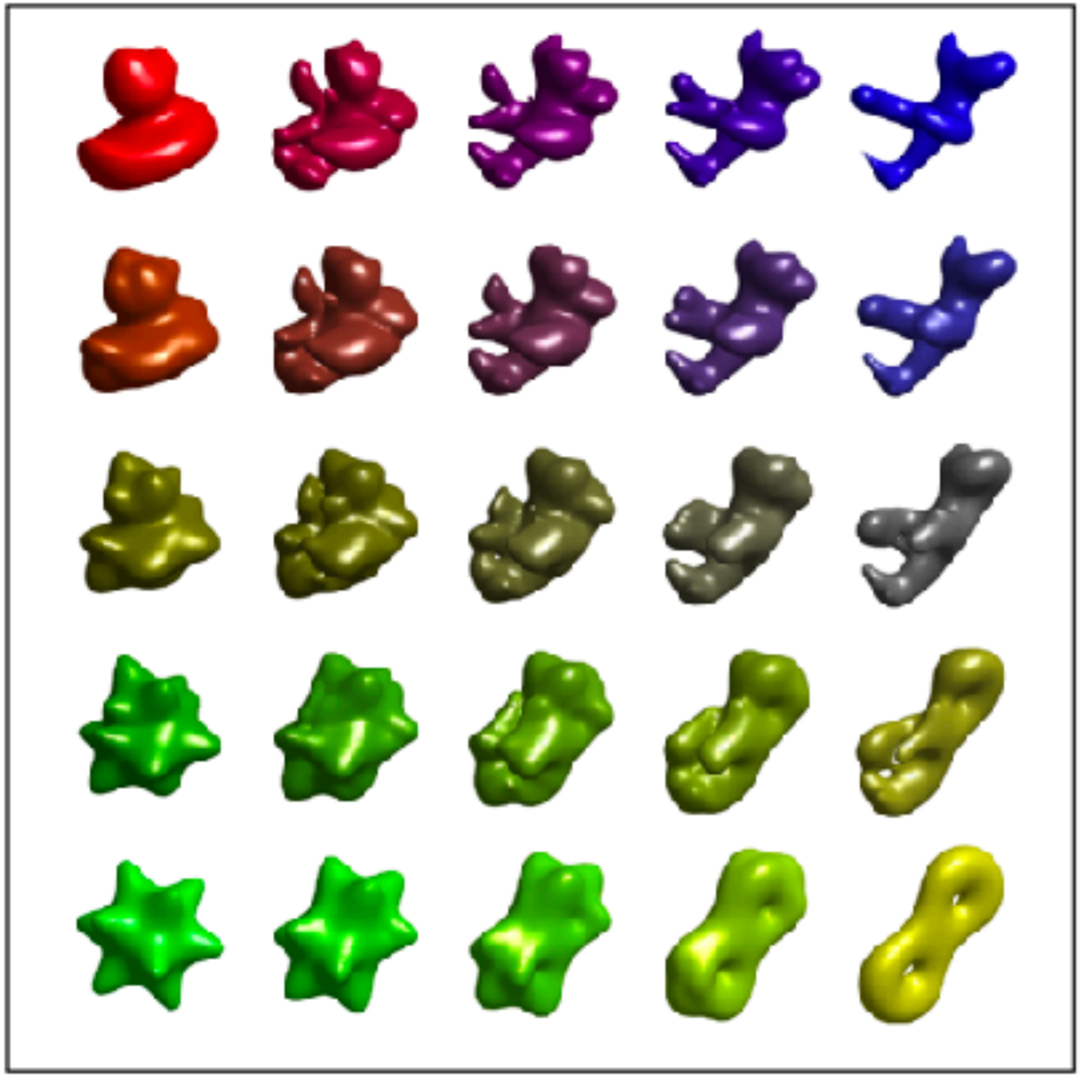
$t = 0$ $t = 0.25$ $t = 0.5$ $t = .75$ $t = 1$

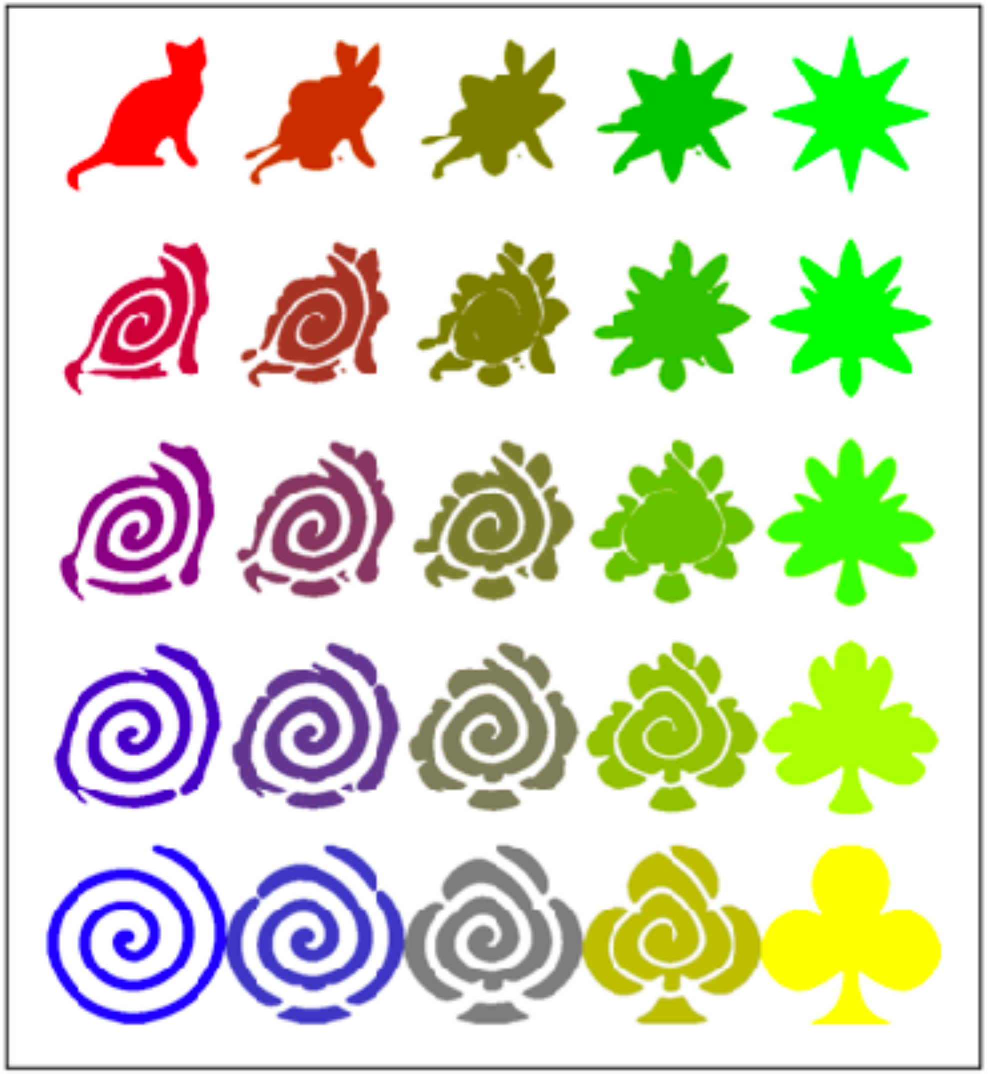
Image Color Palette Equalization





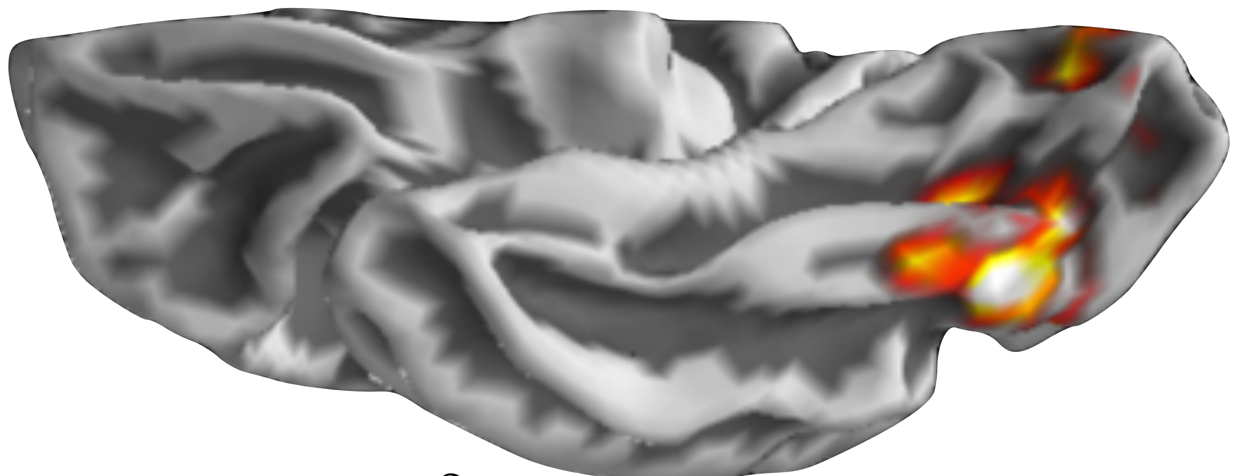
Shape Interpolation



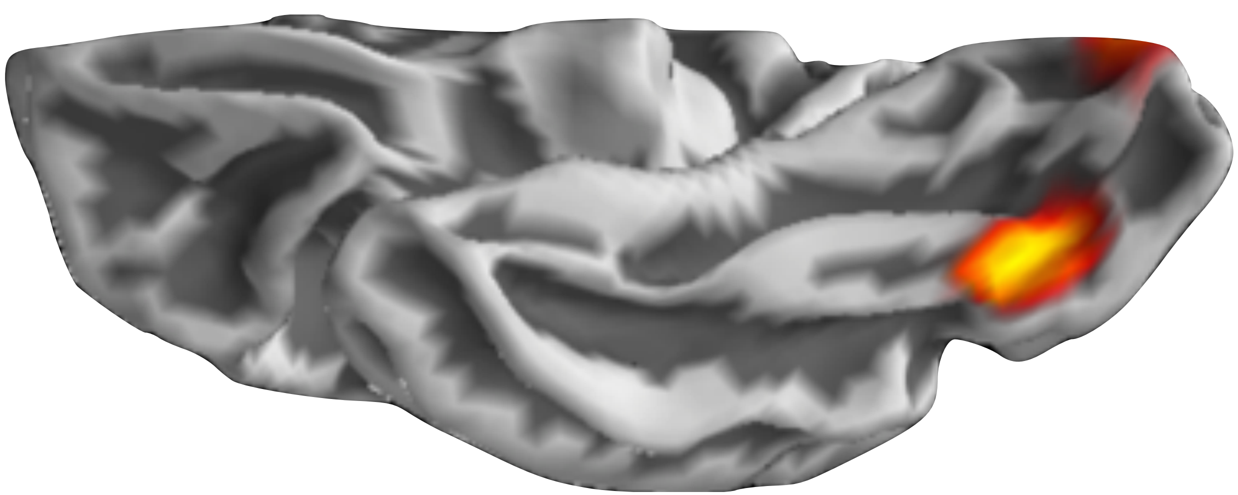


MRI Data Procesing [with A. Gramfort]

Ground cost $c = d_M$: geodesic on cortical surface M.



L^2 barycenter



W_2^2 barycenter

Gradient Flows Simulation

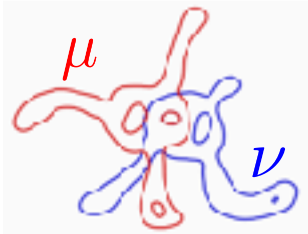
https://www.youtube.com/watch?v=tDQw21ntR64
Tim Whittaker (New Zealand)

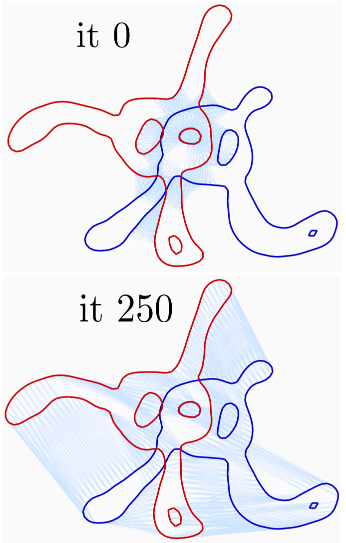
OT Loss for Diffeomorphic Registration

Joint work with J. Feydy, B. Charier, F-X. Vialard.

Shape registration: $\min_{\varphi \text{ diffeo}} D(\varphi(\mu), \nu) + R(\varphi)$

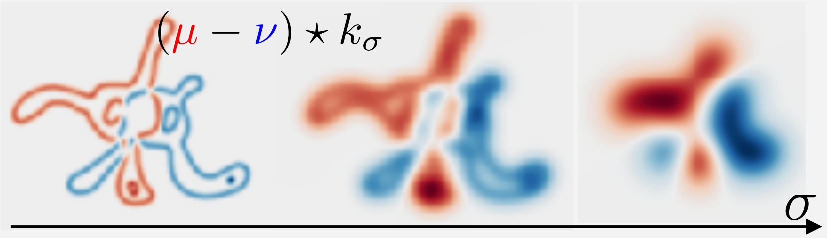
loss — regularity





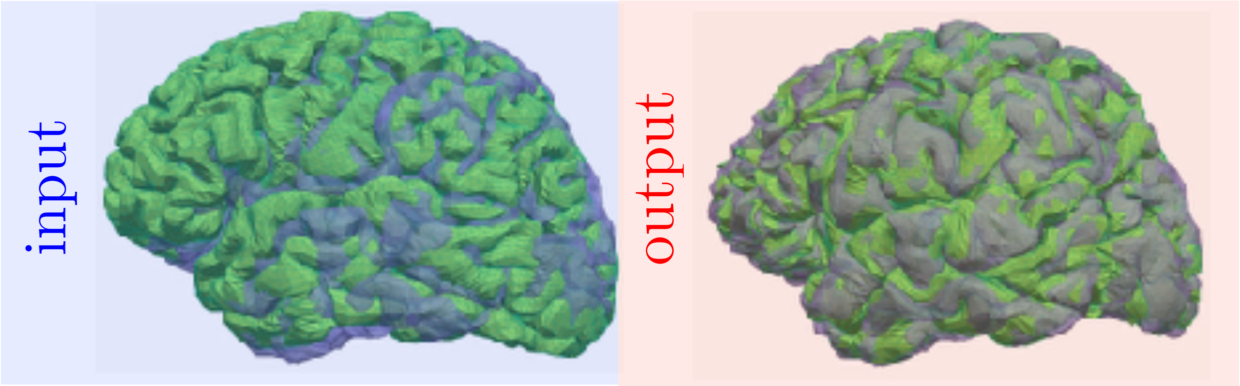
Hilbertian loss (MMD/RKHS):

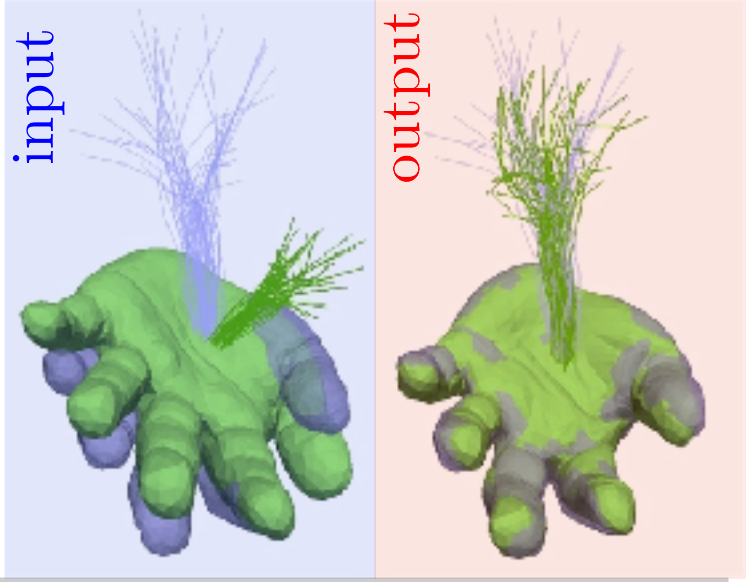
$$D(\mu, \nu) = \|k_\sigma \star (\mu - \nu)\|^2_{L^2}$$



Sinkhorn divergence:

$$D(\mu, \nu) = \bar{W}_\varepsilon(\mu, \nu)$$





→ Do not use OT for registration … but as a loss.

→ Sinkhorn's iterates "propagate" a small bandwidth kernel.

→ Automatic differentation: game changer for advanced loss and models.

Bag of Words

[Kusner'15] $\mathrm{dist}(D_1, D_2) = W_2(\boldsymbol{\mu}, \boldsymbol{\nu})$

Topic Models

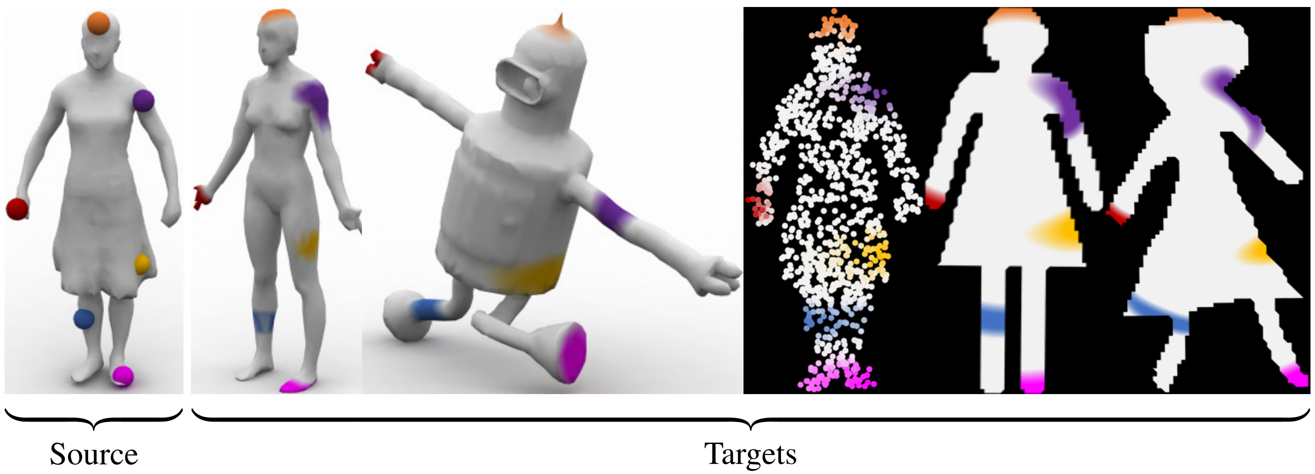
[Rolet'16]

Shapes Analysis with Gromov-Wasserstein

Use T to define registration between:

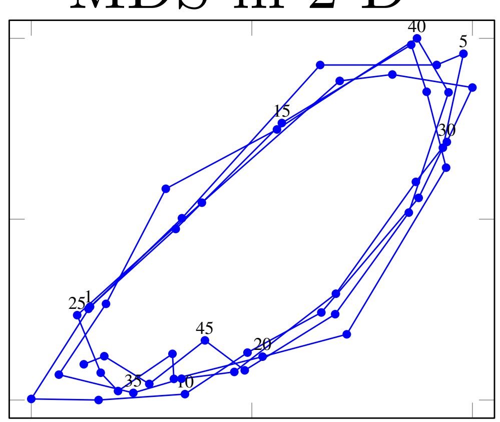
Shape $\longleftrightarrow$ Shape

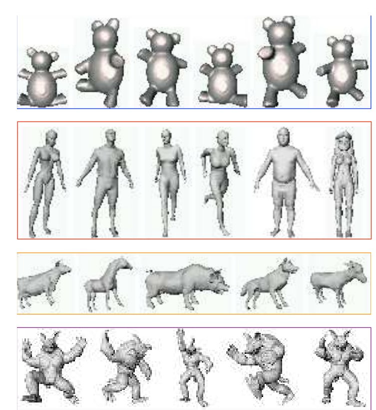
Colors distribution $\longleftrightarrow$ Shape



Shapes $(X_s)_s$ $\rightarrow$ Geodesic distances $d_s = (D_{X_s}(x_i, x_{i'}))_{i,i'}$ $\rightarrow$ GW distances $(\mathrm{GW}_\varepsilon(d_s, d_{s'}))_{s,s'}$ $\rightarrow$ MDS Vizualization

MDS in 2-D





MDS in 3-D